Engineering Notes

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An Inverse Boundary Element Method for Single Component Airfoil Design

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Nomenclature

a,b	$=\xi-$ and $\eta-$coordinates of control point
c	= airfoil chord
d	= half of the element length
k,ϵ,μ	= parameters defining Kármán-Trefftz airfoil
K_{ij}	= element of the influence coefficient matrix
r	= distance from a vortex density contribution to a control point
s	= curvilinear coordinate along the airfoil contour
S	= airfoil perimeter
U_0	= freestream velocity
x,y	= Cartesian coordinates
α	= angle between the x-axis and the freestream
γ	= strength of vortex sheet
ξ,η	= element local Cartesian coordinates
ψ	= stream function
ψ_S	= value of stream function along the airfoil contour

Introduction

An important task in airfoil design is the determination of the airfoil contour corresponding to a prescribed velocity distribution. In the present study, this design problem, also called inverse problem, is considered for single component airfoils in inviscid, incompressible flow. There are several possible approaches to solve the inverse problem; here the stream function method is utilized. In this method, an integral equation is formed for the stream function describing the flow.

The stream function method has been used by several researchers in connection with problems of both analysis and design. Oellers[1] used the method in the analysis of flow around cascades in 1962. Ormsbee and Chen[2] presented a design method in 1972 using a constant strength vorticity element. Later on, variants of this method were developed by Mavriplis,[3] and Kennedy and Marsden.[4]

In this Note, a new method is developed which utilizes a straight line element with linearly varying vortex density. Here the control points are placed exactly on the airfoil contour, and therefore a separate determination of the contour coordinates is eliminated. This makes the new method simpler than the previous approaches.

Received Oct. 6, 1984; revision received Jan. 10, 1985.

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Inverse Calculation Method

Using the stream function method the airfoil in two-dimensional potential flow is replaced by a vortex sheet on the airfoil surface. To satisfy the condition that there should be no flow through the airfoil surface, the total stream function

$$\psi = U_0(y\cos\alpha - x\sin\alpha) + 1/2\pi\oint_S \gamma(s)\ln\left(\frac{r}{c}\right)ds \quad (1)$$

must be constant along the whole surface. For a closed airfoil, the velocity distribution is equal to the vortex distribution.

In the boundary element approach, the integral equation is solved by dividing the vortex sheet into boundary elements that may have varying vortex density. In collocation methods, the values of vortex density are expressed at nodes j and the integral equation is satisfied at an equal number of collocation points i. The integral equation can now be replaced at a specified collocation or control point i by the algebraic equation

$$\psi_S = U_0(y_i\cos\alpha - x_i\sin\alpha) - \sum_j K_{ij}\gamma_j \quad (2)$$

where K_{ij} is the influence coefficient of the vortex density γ_j at node j. The expression for the y coordinate at control point i can be written

$$y_i = \frac{1}{U_0\cos\alpha}\left[\psi_S + \sum_j K_{ij}\gamma_j\right] + x_i\tan\alpha \quad (3)$$

This equation, which presents a formula to solve iteratively the y coordinates of the control points, is common to many inverse calculation methods.

When an element with a constant vortex density is used, the control point is usually placed in the middle of the element. The control point then falls inside the airfoil contour with the element endpoints being on the contour. This raises the problem of determining the relative location of the airfoil contour with respect to the derived control points. Chen[2] utilized a third order Lagrangian interpolation to establish the relation between the airfoil contour and the control points. Kennedy and Marsden[4] preferred a method where the pseudo-control

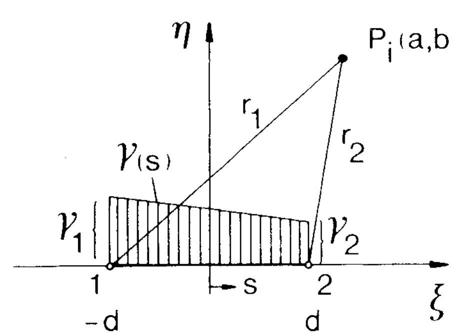

Fig. 1 Linear vortex element in local (ξ,η) coordinate system. P_i is an arbitrary control point.

point at the airfoil trailing edge is used as a starting point for straight line segments passing through the control points. The method is fairly simple but, according to them, contains the risk of producing a sawtooth-shaped airfoil, since small errors can propagate from the trailing edge to the leading edge.

In the present method these difficulties are avoided by using a straight line element with linearly varying vortex density. Both the nodes and the control points are placed at the element endpoints, exactly on the airfoil contour. The influence coefficients of a linear element can be derived by analytic integration.[5] Using the notation of Fig. 1, the general expressions for the influence coefficients of the left-and right-hand nodal vorticities, γ_1 and γ_2, of an element are of the form

$$K'_{i1} = -1/4\pi\Big\{(a+d)\ln r_1 - (a-d)\ln r_2 - 2d + b\Big[\overline{\arctan}\frac{a+d}{b} - \overline{\arctan}\frac{a-d}{b}\Big]\Big\} + 1/4\pi\Big\{\frac{a^2-b^2-d^2}{2d}[\ln r_1 - \ln r_2] - a + ab/d\Big[\overline{\arctan}\frac{a+d}{b} - \overline{\arctan}\frac{a-d}{b}\Big]\Big\} \tag{4}$$

$$K'_{i2} = -1/4\pi\Big\{(a+d)\ln r_1 - (a-d)\ln r_2 - 2d + b\Big[\overline{\arctan}\frac{a+d}{b} - \overline{\arctan}\frac{a-d}{b}\Big]\Big\} - 1/4\pi\Big\{\frac{a^2-b^2-d^2}{2d}[\ln r_1 - \ln r_2] - a + ab/d\Big[\overline{\arctan}\frac{a+d}{b} - \overline{\arctan}\frac{a-d}{b}\Big]\Big\} \tag{5}$$

where definitions

$$r_1 = \sqrt{(a+d)^2 + b^2} \tag{6}$$

$$r_2 = \sqrt{(a-d)^2 + b^2} \tag{7}$$

have been used. Equations (4) and (5) are valid outside the element. When the control point coincides with either of the nodes of the element, the equations reduce to the forms:

$$K'_{11} = K'_{22} = d/4\pi[3 - 2\ln(2d)] \tag{8}$$

$$K'_{12} = K'_{21} = d/4\pi[1 - 2\ln(2d)] \tag{9}$$

With the element influence coefficients of Eqs. (4), (5), (8) and (9), the global influence coefficients, K_{ij}, can be assembled without difficulty.

The y coordinates of the airfoil contour are solved iteratively through Eq. (3) starting from an arbitrary airfoil. During the iteration, the global influence coefficients K_{ij} are calculated for the airfoil from the previous iteration step.

Examples

An example of the convergence of the present design method is given in Fig. 2. This shows the specified velocity distributions on the upper and lower airfoil surfaces, the initial airfoil for starting the iteration, the final airfoil contour after ten iteration cycles and the corresponding velocity distributions. The required computing time is approximately 3.5 s CPU time per iteration cycle on a DEC-20 computer.

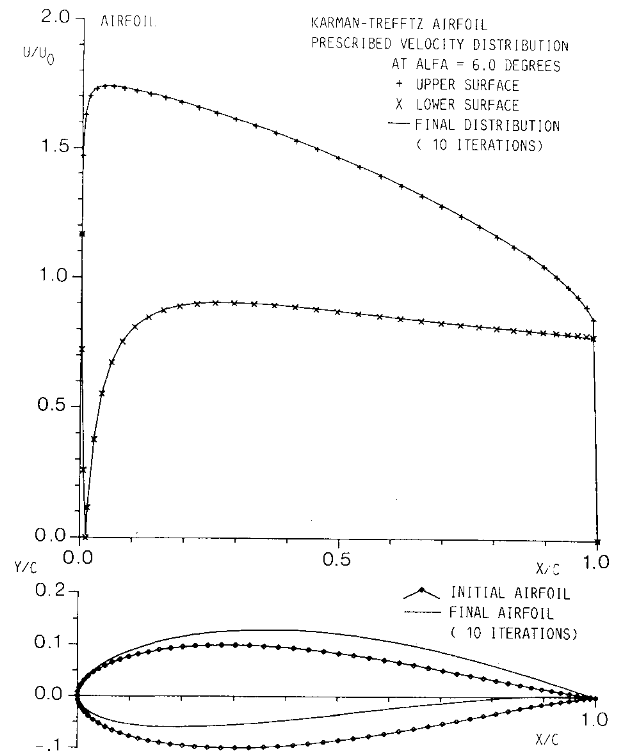

Fig. 2 Determination of the contour of a Kármán-Trefftz airfoil from the prescribed velocity distribution (75 nodes).

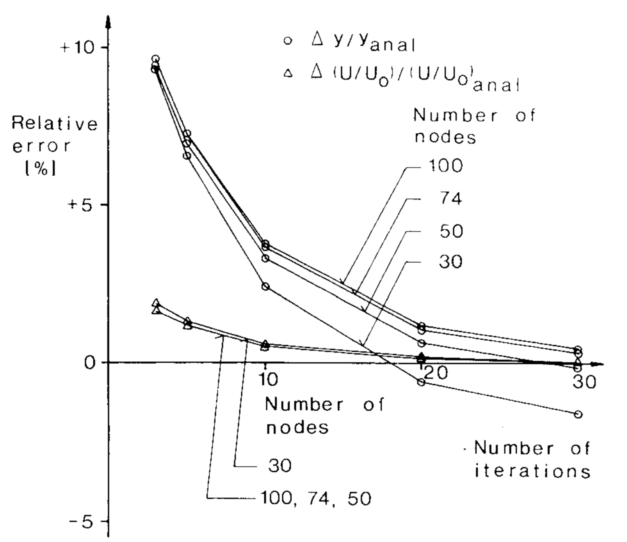

Fig. 3 Relative error of the dimensionless y-coordinate and velocity on the upper surface of a Kármán-Trefftz airfoil at the node at $x/c \approx 0.4$. The final airfoil has parameter values $\epsilon = 0.13$, $k = 1.95$ and $\mu = 0.1$. The initial airfoil has parameter values $\epsilon = 0.15$, $k = 1.9$ and $\mu = 0$.

Previous researchers have used the velocity distribution of the final airfoil as a measure of the convergence of design methods. However, in the opinion of the authors this gives a distorted view of the phenomenon because the element-specific error is not taken into account. The convergence of the velocity and y-coordinate value on the upper surface of the Kármán-Trefftz airfoil of Fig. 2 is shown in Fig. 3 for differing numbers of nodes used. The velocity error moves clearly toward zero as is required in the iteration sheme, whereas the element-specific error causes the $y-$coordinate error to move towards a level other than zero. The final airfoil would probably be too thin in all of the four cases, although the $y-$coordinate error has not changed sign at the two highest numbers of nodes.

Reference 5 includes a computer program for the inverse and analysis problems as well as results of further numerical tests.

Conclusions

An inverse boundary element method for airfoil design has been developed using a straight line element with linearly varying vortex density. The control points are placed exactly on the airfoil contour, making any interpolation or smoothing of the contour during the iteration unnecessary. The new method is more straight-forward than the previous approaches.

It has been shown that when the convergence of a design method is studied, the element-specific error must be taken into account.

Acknowledgments

This work has been supported by the Research Council for Technology of the Academy of Finland.

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